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Few-Body Systems

Similarity Renormalization Group for Few-Body Systems

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Abstract. Internucleon interactions evolved via flow equations yield soft potentials that lead to rapid variational convergence in few-body systems.

The Similarity Renormalization Group (SRG) [1, 2] provides a compelling method for evolving internucleon forces to softer forms by decoupling low- from high-momentum matrix elements [3, 4]. A series of unitary transformations parameterized by s (or $\lambda \equiv s^{-1/4}$) is implemented through a flow equation:

$$H_s = U_s H U_s^\dagger \equiv T_{\rm rel} + V_s \quad \Longrightarrow \quad \frac{dH_s}{ds} = [[G_s, H_s], H_s] \;, \tag{1}$$

where $T_{\rm rel}$ is the relative kinetic energy. Applications to nuclear physics to date in a partial-wave momentum basis have used $G_s = T_{\rm rel}$ [3], so the flow equation for each matrix element is (with $\epsilon_k \equiv \langle k|T_{\rm rel}|k\rangle = \hbar^2 k^2/m$)

$$\frac{d}{ds}\langle k|V_s|k'\rangle = -(\epsilon_k - \epsilon_{k'})^2 \langle k|V_s|k'\rangle + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q)\langle k|V_s|q\rangle\langle q|V_s|k'\rangle \;. \tag{2}$$

The flow of off-diagonal matrix elements is dominated by the first term, which drives them rapidly to zero. This partially diagonalizes the momentum-space potential, leading to decoupling [4]. Pictures showing different initial NN potentials evolving to band-diagonal form can be viewed at the SRG website [5].

In the left panel of Fig. 1, the ^{1}S$_0$ phase shift for the Argonne v_{18} NN potential is shown up to 800 MeV lab energy. The phase shifts for the SRG potential V_s are indistinguishable at any λ because the evolution is exactly unitary at the two-body level. To test decoupling, the original and evolved SRG potential (to $\lambda = 2\,\text{fm}^{-1}$) are smoothly set to zero for momenta above $k_{\rm max} = 2.2\,\text{fm}^{-1}$. The SRG phases are unchanged up to the corresponding $E_{\rm lab}$, so high momenta are *not* needed. The AV18 phases are completely changed because even low-energy observables have contributions from high momentum, which has led to the misconception that high-energy phase shifts are important for nuclear structure [4].

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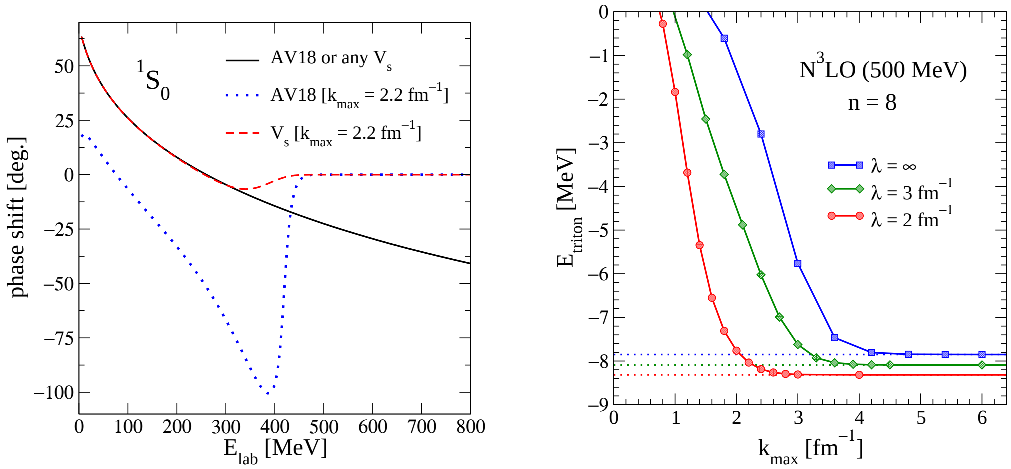

Figure 1. Left: Decoupling in the ${}^1\mathrm{S}_0$ phase shift for the Argonne v_{18} NN potential [4]. Right: Decoupling in the triton with the N^{3}LO chiral EFT potential of Entem and Machleidt [6].

A similar story for the triton ground-state energy with a chiral EFT N^{3}LO potential is seen in the right panel, which shows the energy as a function of $k_{\max}$. Because of decoupling, the full answer for smaller λ is reached for lower $k_{\max}$. A consequence is faster convergence in variational and *ab initio* few-body calculations, as shown in the left panel of Fig. 2 for two N^{3}LO potentials, where the energy is plotted against the size of a harmonic oscillator basis. A complete study with NN potentials in the no-core shell model is given in ref. [6].

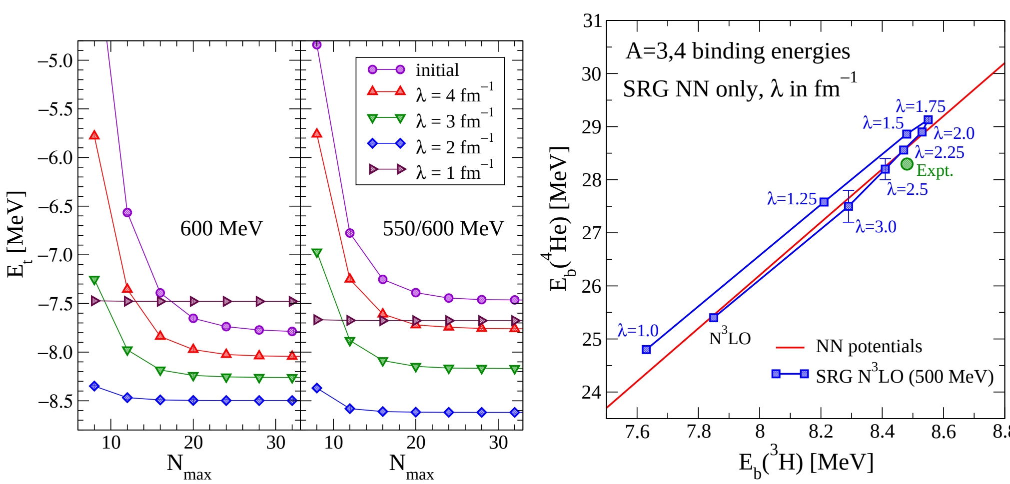

Figure 2. Left: Convergence in the triton [4]. Right: Tjon line traced out by SRG-evolved NN potentials, labeled by λ [6].

The commutators in Eq. (1) imply that the evolving Hamiltonian will have many-body interactions to all orders (i.e., insert second-quantized operators). Thus there will always be a truncation and the evolution will only be approximately unitary. The present calculations evolve only the NN part, which explains why different converged triton energies are seen in Fig. 2. This is a controlled

approximation in the range of λ for which the variation is comparable to the truncation error inherent in the initial EFT Hamiltonian. The variation is seen to be natural in Fig. 2 and the left panel of Fig. 3, which also shows the improved convergence (decreasing error bars) for smaller λ [6].

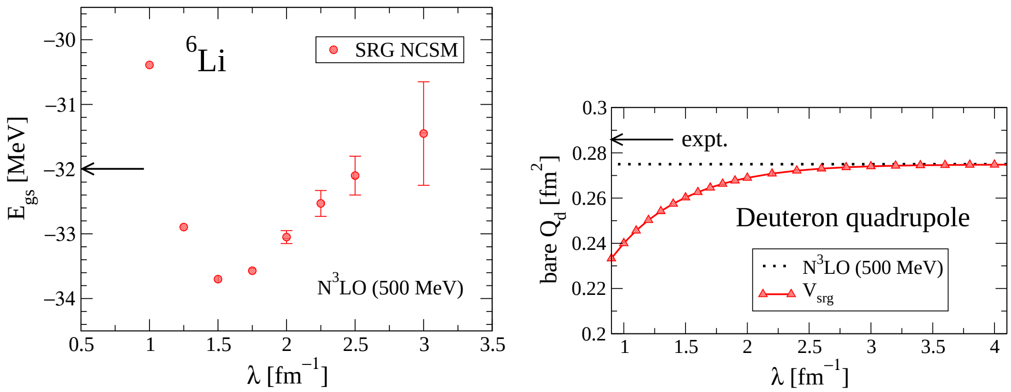

Figure 3. Left: Ground-state energy in ^{6}Li vs. λ [6]. Right: Running of the bare (unevolved) deuteron quadrupole moment.

Including the 3N interaction is essential for nuclear structure. The SRG evolution only modifies the short-distance part of the potential or operators. This is illustrated by the weak running of the bare quadrupole moment in the right panel of Fig. 3. Since the chiral EFT 3N force will be modified only at short distance, a good first approximation should be to simply re-fit its two short-distance coefficients at each λ. In parallel, we are implementing the 3N evolution by applying Eq. (1) in the three-particle space, which does not require solving the full three-nucleon problem [3]. The evolution of the NN interaction is independent of spectators and the equation for the 3N interaction has no disconnected pieces. A model calculation that introduces a diagrammatic treatment is described in ref. [7].

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